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F-term hybrid inflation, the η-problem and extra dimensions

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Abstract: F-term hybrid inflation models in the context of supergravity generically have corrections to the inflaton mass squared of the order of H^2 (the η-problem). In addition they have a problem with large deviations of the spectrum of density perturbations from scale-invariance due to non-renormalizable corrections to the superpotential. Here we show that an increase of the expansion rate at large energy densities relative to that in conventional $d = 4$ cosmology, as suggested by brane scenarios, can naturally solve the η problem. For the case of a ρ^2 squared correction to H^2 suppressed by the brane tension this requires that the $d = 5$ Planck mass satisfies $M_5 \lesssim 10^{16}$ GeV. In addition, the scale-invariance of the density perturbation spectrum can be generally protected from Planck-suppressed superpotential corrections if $M_5 \lesssim 10^{10}$ GeV. Therefore brane cosmologies which produce an enhanced expansion rate at large energy densities are favoured by SUSY hybrid inflation.

Keywords: Extra Large Dimensions, Supergravity Models, Physics of the Early Universe.

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1. Introduction

Hybrid inflation models [1] are a favoured class of inflation model, being able to account for both the flatness of the inflaton potential and the rapid inflaton oscillations which end inflation, without the need for very small couplings. Supersymmetric (SUSY) versions are classified as F- [2, 3] or D-term [4] hybrid inflation models, depending on the origin of the energy density driving inflation. However, once globally SUSY models are generalized to supergravity (SUGRA), a number of problems arise. For the case of F-term hybrid inflation models, there are corrections to the inflaton mass squared of the order of the expansion rate squared arising from SUGRA corrections [5, 3]. Although it is possible to eliminate such corrections by choosing the Kähler potential carefully, such as having a minimal form [3] or a Heisenberg symmetry [6], in most models such corrections result in an inflaton potential which cannot produce successful slow-roll inflation (the η-problem). In addition, SUSY hybrid inflation models (both F- and D-term) have a problem with deviations of the inflaton potential from flatness due to Planck-suppressed non-renormalizable corrections to the superpotential. This arises due to the large initial inflaton expectation value, typically close to the Planck scale, required in order to achieve sufficient inflation [7, 8].

All this assumes the standard $d = 4$ cosmology based on the homogeneous and isotropic Friedmann-Robertson-Walker (FRW) metric. In the past few years there has been considerable interest in the possibility that spacetime may have more than four dimensions [9]–[11], which is possible if conventional matter is confined to a $d = 4$ brane. For example, in the Randall-Sundrum (RS) brane scenario with a single brane and one uncompactified extra dimension (RS2) [11], the brane has a positive tension and the $d = 5$ bulk a negative cosmological constant such that the sum cancels out on the brane, giving zero (or very small, if the cancellation is not exact) cosmological constant in the $d = 4$ Universe. Conventional gravity is obtained on the brane by localizing the massless graviton mode via the near-brane geometry. The cosmology of this scenario is no longer described by the $d = 4$ FRW metric but instead one must solve the $d = 5$ Einstein equations for the evolution of the full $d = 5$ metric. The solution of the Einstein equations in the RS2 scenario including matter trapped on the brane results in a different evolution for the $d = 4$ expansion rate H as a function of the energy density ρ as compared with conventional $d = 4$ cosmology [12, 13], such that at large energy density H is proportional to ρ rather than $\rho^{1/2}$. It follows that inflationary cosmology has to be reconsidered in the RS2 brane scenario. This has recently been done for the case of chaotic inflation, where it was found that the brane has the advantage of reducing the magnitude of the inflaton field during inflation to below the Planck scale, so evading dangerous Planck-suppressed non-renormalizable corrections to the inflaton potential [14, 15].

However, it has been shown that the single-brane RS2 scenario is not compatible with SUSY [16]. Nevertheless, we generally expect to find corrections to the expansion rate as a function of the energy density in brane scenarios [17, 18].[1] Generalized brane corrections of the form $H^2 = A\rho + B\rho^n$ have recently been suggested in order to account for the observed accelerated expansion of the Universe [19]. Any relation between H^2 and ρ is in principle possible in brane cosmology, depending on the bulk stress-energy tensor and boundary conditions [18], the simplest case being that of a cosmological constant in the bulk and a cancelling positive tension on the brane, which results in a ρ^2 correction suppressed by the brane tension [17]. In order to explore the possible advantages for SUSY inflation of an enhanced expansion rate at large energy densities, in the following we will consider the case of $N = 1$ $d = 4$ SUGRA inflation but with a ρ^2 correction to the expansion rate suppressed by the brane tension, using the RS2 correction as a specific example. We will refer to this as the "brane-inspired scenario". (It should again be emphasized that the RS2 model does not apply in the SUSY case; we use the Friedmann equation of the RS2 model only as an example of what might arise in brane cosmology.) In particular we will consider the case of F-term hybrid inflation. We will show that the η problem can be easily and naturally solved in this scenario and, in addition, that the problem of corrections to the inflaton superpotential may be solved if the fundamental $d = 5$ Planck mass M_5 is sufficiently small.

The letter is organized as follows. In section 1 we discuss the F-term hybrid inflation model in the brane-inspired scenario. In section 2 we review the η-problem and its solution in the brane-inspired scenario. In section 3 we consider the problem of non-renormalizable corrections to the superpotential and its solution in the brane-inspired scenario. In section 4 we present our conclusions.

2. F-term hybrid inflation in the brane-inspired scenario

The F-term hybrid inflation model is described by the superpotential [2]

$$W = S\left(\kappa\overline{\psi}\psi - \mu^2\right). \tag{2.1}$$

[1] It has also been noted that, due to higher-derivative corrections to the gravitational action, the ρ^2 correction to H^2 must probably be considered as the first term in an expansion in energy density suppressed by powers of the brane tension, in which case we cannot be sure of the form of H^2 at large energy densities in the absence of a full quantum theory of gravity (even in the case of the RS2 scenario) [17].

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The tree-level scalar potential in the global SUSY limit is then

$$V = |\kappa\overline{\psi}\psi - \mu^2|^2 + \kappa^2|S|^2\left(|\overline{\psi}|^2 + |\psi|^2\right). \tag{2.2}$$

We can take the inflaton field to be real and positive. If $s > s_c \equiv \sqrt{2}\mu/\kappa^{1/2}$ (where $\langle S\rangle = s/\sqrt{2}$) then at the minimum of V as a function of s we find $\psi = \overline{\psi} = 0$ and so the tree-level potential as a function of s is flat, $V(s) = \mu^4$. Including the one-loop correction due to ψ and $\overline{\psi}$ loops results in the potential (for s^2 large compared with s_c^2) [2]

$$V(s) \approx \mu^4\left(1 + \frac{\kappa^2}{16\pi^2}\ln\left(\frac{s}{Q}\right)\right), \tag{2.3}$$

where Q is a renormalization scale. The slow-roll equation for s is then

$$3H\dot{s} = -\frac{\partial V}{\partial s} = -\frac{\kappa^2\mu^4}{16\pi^2 s}. \tag{2.4}$$

The Friedmann equation for the $d = 4$ expansion rate H in the RS2 brane scenario is given by [12, 13]

$$H^2 = \frac{\Lambda_4}{3} + \left(\frac{8\pi}{3M_4^2}\right)\rho + \left(\frac{4\pi}{3M_5^3}\right)^2\rho^2 + \frac{\epsilon}{a^4}, \tag{2.5}$$

where Λ_4 is the $d = 4$ cosmological constant, ϵ is due to the influence of graviton modes in the bulk and M_4 and M_5 are the $d = 4$ and $d = 5$ Planck masses respectively. For the RS2 brane scenario M_5 must be greater than 10^8 GeV in order that gravity is not modified on length scales greater than the experimental limit of about 1mm [14]. (We will see that M_5 is generally larger than this in following.) The ϵ term is diminished during inflation by expansion and may be neglected, as may the small $d = 4$ cosmological constant. Therefore during inflation in the RS2 scenario

$$H^2 = \left(\frac{4\pi}{3M_5^3}\right)^2\rho\,(\rho + \rho_c) \equiv \frac{8\pi\rho}{3M_4^2}\left(1 + \frac{\rho}{2\tau}\right), \tag{2.6}$$

where $\tau = 48\pi M_5^6/M_4^2$ is the 3-brane tension and

$$\rho_c = \frac{3}{2\pi}\frac{M_5^6}{M_4^2}. \tag{2.7}$$

H^2 is dominated by the ρ^2 term if $\rho \approx \mu^4 > \rho_c$. We will assume this form for H^2 in the following as a specific example of a brane-generated ρ^2 correction to H^2 suppressed by the 3-brane tension.

Using H^2 in eq. (2.6), the solution to the slow-roll equation eq. (2.4) is given by

$$s^2 - s_c^2 = \frac{N\kappa^2}{24\pi^2\,(\rho + \rho_c)}\left(\frac{3M_5^3}{4\pi}\right)^2, \tag{2.8}$$

where N is the number of e-foldings of inflation as s rolls from s to s_c (assuming H is approximately constant during inflation). The adiabatic density perturbations evolve as in conventional $d = 4$ cosmology [14, 20], with $\zeta \propto \delta\rho/(\rho+p)$ conserved on superhorizon scales, such that $\delta\rho/\rho$ for a density perturbation mode re-entering the horizon is proportional to $\delta\rho/\dot{\phi}^2$ on horizon crossing during inflation. During slow-rolling the COBE normalization of the density perturbation is given by [21]

$$\delta_H \equiv \frac{3}{5\pi}\frac{H^3}{V'} = \delta_{H\ COBE} \equiv 1.91 \times 10^{-5}\,, \tag{2.9}$$

where δ_H is the density perturbation spectrum at the present Hubble scale [21]. COBE normalization then implies that

$$\mu = \left(\frac{15}{256\pi^2}\right)^{1/6}\left(\frac{24}{N}\right)^{1/12}\kappa^{1/6}\delta_{H\ COBE}^{1/6}\left(1+\frac{\rho_c}{\rho}\right)^{-1/6} M_5\,. \tag{2.10}$$

Thus

$$\mu = 0.065\kappa^{1/6}\left(\frac{50}{N}\right)^{1/12}\left(1+\frac{\rho_c}{\rho}\right)^{-1/6} M_5\,, \tag{2.11}$$

where $N \sim 50$ is typical of the number of e-foldings at which COBE length scales exit the horizon.

In addition to imposing COBE normalization we will assume that s^2 is large compared with s_c^2 during inflation, in order that eq. (2.3) is valid. With s_o denoting the inflaton field when the observable Universe leaves the horizon ($N \sim 50-60$), this requires that

$$s_o^2 \approx \frac{N\kappa^2}{24\left(\rho+\rho_c\right)\pi^2}\left(\frac{3M_5^3}{4\pi}\right)^2 \gg s_c^2\,. \tag{2.12}$$

This is satisfied if

$$\mu < \left(\frac{N\kappa^3}{48\pi^2}\right)^{1/6}\left(\frac{3M_5^3}{4\pi}\right)^{1/3}\left(1+\frac{\rho_c}{\rho}\right)^{-1/6}\,, \tag{2.13}$$

with $s_c^2/s_o^2 \propto \mu^6$.

Finally, in order to have a scenario that deviates from conventional $d=4$ F-term hybrid inflation we have to impose that $\rho > \gamma\rho_c$ during inflation (γ is a constant which will be useful later; for now we may take $\gamma = 1$), which requires that

$$M_5 < \left(\frac{2\pi\rho M_4^2}{3\gamma}\right)^{1/6}\,. \tag{2.14}$$

We can combine these conditions to obtain constraints on the coupling κ and the $d=5$ Planck mass M_5. The COBE normalization, eq. (2.11), combined with the condition for $s_o^2 \gg s_c^2$, eq. (2.13), implies the M_5 independent condition

$$\kappa > 3.5 \times 10^{-3}\left(\frac{50}{N}\right)^{3/4}\,. \tag{2.15}$$

The condition $\rho > \gamma\rho_c$, eq. (2.14), combined with the COBE normalization, eq. (2.11), implies that

$$\gamma^{1/2}M_5 < 7.4 \times 10^{16}\kappa^{1/3}\left(\frac{50}{N}\right)^{1/6}\ \mathrm{GeV} \equiv M_{5\ c}\,, \tag{2.16}$$

(where for $M_5 \geq M_{5\ c}$ we have conventional $d=4$ cosmology during inflation). Thus from eq. (2.15) and eq. (2.16), with $\gamma = 1$ and κ of the order of 1, we see that successful F-term hybrid inflaton with the expansion rate dominated by brane effects can be naturally achieved if $M_5 \lesssim 10^{16-17}$ GeV.

3. η-problem in the brane-inspired scenario

In SUGRA models the F-term contribution to the scalar potential is given by [8]

$$V = F^2 - \frac{3}{M^2} e^{K/M^2} |W|^2 \,, \tag{3.1}$$

where W is the superpotential, K is the Kähler potential, $M = M_4/\sqrt{8\pi}$ is the SUGRA mass scale and

$$F^2 = e^{K/M^2} \left(W_m + \frac{W K_m}{M^2} \right)^* K^{m^* n} \left(W_n + \frac{W K_n}{M^2} \right) , \tag{3.2}$$

where $W_m = \partial W / \partial \phi^m$ and $K^{m^* n}$ is the inverse of K_{nm^*}. For example, with the superpotential given by eq. (2.1) and with Kähler potential for the inflaton $K = cS^* S$ (where c is a dimensionless constant and so expected to be of the order of 1), corresponding to inflaton kinetic terms $K_{mn^*} \partial_\mu \phi^{n^*} \partial^\mu \phi^m = (c/2) \partial_\mu s \partial^\mu s$, the SUGRA scalar potential for $s > s_c$ is given by

$$V = V_{\rm susy} + \frac{(1-c)}{c} \frac{\mu^4 s^2}{M^2} + \left(\frac{\mu^4 s^4}{M^4} \right) , \tag{3.3}$$

where $V_{\rm susy}$ is the global SUSY scalar potential and we have rescaled s, μ and κ to have canonical kinetic terms for the inflaton and to match the scalar potential in the global SUSY limit ($M \to \infty$) to that given in eq. (2.2). Thus except in the case of c equal to 1 (corresponding to a minimal Kähler potential), the inflaton typically gains a mass squared correction $\sim \pm \mu^4/M^2$. In the case of conventional $d = 4$ cosmology, the expansion rate during inflation is $H^2 = \mu^4/3M^2$. Therefore the magnitude of the mass squared correction is of the order of H^2. However, it should be emphasized this is *essentially coincidental*. For the case of the brane-inspired scenario, the mass squared correction remains $\sim \pm \mu^4/M^2$ but, for energy density $\rho > \rho_c$, H^2 from eq. (2.6) is now larger than $\mu^4/3M^2$ by a factor ρ/ρ_c. Thus so long as ρ/ρ_c is sufficiently large (greater than about 300), the mass squared correction to the inflaton potential will be less than $0.1H$ and so will not play a significant role in the dynamics of the inflaton. (The magnitude of η-parameter (defined in eq. (4.3) below) due to the mass squared correction is then $\lesssim 1/300$ and so completely negligible.) From eq. (2.16) with $\gamma = 300$ we see that this requires that $M_5 \lesssim 10^{16}$ GeV. Therefore so long as the scale of gravity in the bulk is sufficiently small, the η-problem of F-term hybrid inflation models is solved in the brane-inspired scenario.

4. Non-renormalizable superpotential corrections

In conventional $d = 4$ cosmology there is a second problem for SUSY hybrid inflation models associated with the large initial value of the inflaton field required to account for sufficient inflation [7]. From eq. (2.8), if $\rho < \rho_c$, corresponding to conventional $d = 4$ cosmology during inflation, then the value of s_o (for s_o^2 large compared with s_c^2), is given by

$$s_o = \frac{\kappa N^{1/2} M}{2\sqrt{2}\pi} \,. \tag{4.1}$$

Thus in conventional $d = 4$ cosmology and with κ of the order of 1, the initial expectation value of the inflaton is of the order of M. Since in SUGRA there is no reason to restrict the superpotential to renormalizable terms, we expect in general to find Planck-suppressed non-renormalizable superpotential terms of the form $\lambda_n S^n/n!M^{n-3}$ (where we include a symmetry factor $n!$). This results in a correction to the inflaton potential

$$\delta V = \frac{\lambda_n^2 |S|^{2(n-1)}}{((n-1)!)^2 M^{2(n-3)}} \,. \tag{4.2}$$

Since in $d = 4$ cosmology s is typically not very small compared with M, this results in a large deviation of the spectrum of density perturbation from scale-invariance [7]. The spectral index is given by $n = 1 + 2\eta - 6\epsilon$, where in general during slow-rolling

$$\eta = \frac{1}{3}\frac{V''}{H^2}\,, \qquad \epsilon = \frac{1}{3}\frac{V'^{\,2}}{H^3}\frac{\partial H}{\partial V}\,, \tag{4.3}$$

with prime denoting $\partial/\partial s$. (Observationally $|n-1| \lesssim 0.1$ [22].) For the 1-loop correction, eq. (2.3), $\eta = -1/2N$ and $|\epsilon/\eta| \approx \kappa^2/16\pi^2 \ll 1$. Assuming that the contribution of the superpotential correction δV to V'' dominates that of the 1-loop correction, η is then given by

$$\eta = \frac{2}{3}\left(\frac{3M_5^3}{4\pi}\right)^2 \frac{(n-1)(2n-3)\tilde{\lambda}_n^2 s^{2n-4}}{\rho\,(\rho+\rho_c)\, M^{2(n-3)}}\,, \tag{4.4}$$

where $\tilde{\lambda}_n = \lambda_n/(2^{(n-1)/2}(n-1)!)$. (For V', V'' dominated by δV, $|\epsilon/\eta| \approx \delta V/V < 1$. Therefore η gives the dominant contribution to the index.) Imposing the constraint that $|2\eta| \lesssim |\Delta n|$ then implies

$$\left(\frac{s}{M}\right)^{2n-4} < \frac{3}{4}\left(\frac{4\pi}{3M_5^3}\right)^2 \frac{\rho(\rho+\rho_c)\Delta n}{(n-1)(2n-3)\tilde{\lambda}_n^2 M^2}\,. \tag{4.5}$$

The largest non-renormalizable correction corresponds to $n = 4$. From eq. (2.12), the value of s_o for $\rho > \rho_c$ and $s_o^2 \gg s_c^2$ is

$$s_o^2 = \frac{N\kappa^2}{24\pi^2\rho}\left(\frac{3M_5^3}{4\pi}\right)^2\,. \tag{4.6}$$

Thus with $s = s_o$ in eq. (4.5) we obtain for $n = 4$ an upper limit on M_5,

$$M_5 < 1.9\times 10^{10}\left(\frac{1}{\tilde{\lambda}_4\kappa^{2/3}}\right)\left(\frac{50}{N}\right)^{5/3}\left(\frac{\Delta n}{0.1}\right)^{1/2}\ \mathrm{GeV}\,. \tag{4.7}$$

Thus for all $\kappa \lesssim 1$, if $M_5 \lesssim 10^{10-11}$ GeV then *all* non-renormalizable corrections are compatible with scale-invariance of the spectrum of density perturbations. Note that from COBE normalization, eq. (2.11), the corresponding scale of the energy density driving inflation, $V^{1/4} = \mu$, is then $\mu \lesssim 10^{9-10}$ GeV. Therefore the thermal gravitino upper bound on the reheating temperature, $T_R \lesssim 10^8$ GeV [25], should be easier to satisfy in this case.

In general, even if Planck-suppressed non-renormalizable superpotential corrections are acceptable in the brane-inspired scenario, renormalizable S^2 and S^3 terms must still

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be eliminated. Although it is possible to eliminate all superpotential corrections (both renormalizable and non-renormalizable) via a global R-symmetry, it has been argued that global symmetries are generally broken by wormhole [23] or string [24] effects, leaving only discrete symmetries as a possibility for eliminating unwanted superpotential terms. In this case it is possible that at least some non-renormalizable superpotential corrections will be allowed by the discrete symmetry. Therefore the ability to tolerate unsuppressed non-renormalizable corrections may be a significant advantage for enhanced expansion at large energy densities.

5. Conclusions

We have discussed how F-term hybrid inflation in the context of $N = 1$ supergravity models may be modified advantageously via an enhanced expansion rate as a function of ρ, such as may occur in brane cosmology. As a specific model of a ρ^2 correction to H^2 suppressed by powers of the 3-brane tension we assumed the form of the Friedmann equation obtained in the (non-SUSY) RS2 scenario. (This is used here only to model the form of expansion rate that might plausibly be obtained in brane cosmology and in order to illustrate the advantages of an enhanced expansion rate for F-term inflation.) If the $d = 5$ Planck scale M_5 is less than about 10^{16} GeV then SUGRA corrections to the inflaton mass are less than $0.1H$, so solving the η problem. In addition, if $M_5 \lesssim 10^{10-11}$ GeV, then Planck-suppressed non-renormalizable superpotential corrections do not cause an unacceptable deviation of the density perturbation spectrum from scale-invariance. In this case the energy density during inflation satisfies $\rho^{1/4} \lesssim 10^{9-10}$ GeV, making it easier for the reheating temperature to satisfy the thermal gravitino upper bound. Although we have discussed a specific form for the enhanced expansion rate, we expect that the benefits for SUSY hybrid inflation will be obtained for enhanced expansion rates in general. Thus an increased expansion rate as compared with conventional $d = 4$ cosmology, such as may occur in brane cosmology, is favoured by SUSY F-term hybrid inflation. This may serve as a motivation for a finding a SUSY brane cosmology with a suitably enhanced expansion rate at large energy densities.

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